Math 113-008, Exam 2
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Name ____________________
Row____________________

1. (80%) Evaluate the following integrals. If an improper integral diverges, so state. Show your work.

(a) $\displaystyle\int \frac{x^2+1}{x+1}\,dx = \int \left(x+1+\frac{2}{x+1}\right)\,dx = \frac{x^2}{2}+x+2\ln|x+1|+C$

(b) $\displaystyle\int \sin 2x \sin 3x\,dx$

$$\sin A \sin B = \frac{\cos(A-B)-\cos(A+B)}{2}$$

$$\int (\sin 2x \sin 3x)\,dx = \frac{1}{2}\int (\cos(-x)-\cos(5x))\,dx = \frac{1}{2}\int (\cos(x)-\cos(5x))\,dx$$

$$= \frac{\sin(x)}{2} - \frac{\sin(5x)}{10} + C$$

(c) $\displaystyle\int_0^{\pi/4} \tan^2 x \sec^4 x\,dx = \int_0^{\pi/4} \tan^2 x \sec^2 x \sec^2 x\,dx = \int_0^{\pi/4} \tan^2 x(1+\tan^2 x)\sec^2 x\,dx$

$$= \int_0^{\pi/4} (\tan^2 x + \tan^4 x)\sec^2 x\,dx = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$$

(d) $\displaystyle\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\,dx$

Let $x = 2\sin\theta$ and $dx = 2\cos\theta d\theta$. For $0 < x < 2$, we get $0 < \theta < \dfrac{\pi}{2}$.

$$\int_0^{\pi/2} \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} 2\cos\theta\,d\theta = 4\int_0^{\pi/2} \sin^2\theta\,d\theta = 2\int_0^{\pi/2} (1-\cos(2\theta))\,d\theta$$

$$= (2\theta - \sin(2\theta))|_0^{\pi/2} = \pi$$

(e) $\displaystyle\int_0^4 \sqrt{4x - x^2}\,dx = \int_0^4 \sqrt{4 - (x^2 - 4x + 4)}\,dx = \int_0^4 \sqrt{4 - (x-2)^2}\,dx = 2\pi$

(f) $\displaystyle\int \frac{4x}{(x-1)^2(x+1)}\,dx$

Use partial fractions. $\displaystyle\frac{4x}{(x-1)^2(x+1)} = \frac{1}{x-1} + \frac{2}{(x-1)^2} - \frac{1}{x+1}$

$$\int \frac{4x}{(x-1)^2(x+1)}\,dx = \ln|x-1| - \ln|x+1| - \frac{2}{x-1} + C$$

(g) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_0^{\infty} \frac{dx}{1+x^2} = \tan^{-1} x_{-\infty}^{0} + \tan^{-1} x_0^{-\infty} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

(h) $\displaystyle\int_0^{\infty} \frac{\sqrt{x}}{1+x}\,dx = \int_0^1 \frac{\sqrt{x}}{1+x}\,dx + \int_1^{\infty} \frac{\sqrt{x}}{1+x}\,dx$

For $x \geq 1$, $\displaystyle\frac{\sqrt{x}}{1+x} \geq \frac{1}{1+x}$. But $\displaystyle\int_1^{\infty} \frac{1}{1+x}\,dx = \ln(1+x)|_1^{\infty}$ diverges. So the integral diverges by the comparison test.

2. (6%) Use the integral definition of $\ln x$ from Appendix G and Simpson's Rule with $n = 4$ to approimate $\ln 5$. You do not need to simplify your expression for the answer.

$$\ln 5 = \int_1^5 \frac{1}{t} dt$$

$$\Delta x = \frac{5-1}{4} = 1$$

$$\ln 5 = \int_1^5 \frac{1}{t} dt \approx \frac{1}{3}[\frac{1}{1} + 4\frac{1}{2} + 2\frac{1}{3} + 4\frac{1}{4} + \frac{1}{5}$$

3. (8%) Find the length of the curve $y = x^2 - \frac{1}{8}\ln x$ for $1 \leq x \leq 3$.

$$\frac{dy}{dx} = 2x - \frac{1}{8x}$$

$$\left(\frac{dy}{dx}\right)^2 = 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$$

Length equals $\int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_1^3 \sqrt{1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}}\, dx$

$$= \int_1^3 \sqrt{4x^2 + \frac{1}{2} + \frac{1}{64x^2}}\, dx = \int_1^3 \sqrt{4x^2 + \frac{1}{2} + \frac{1}{64x^2}}\, dx = \int_1^3 \sqrt{\left(2x - \frac{1}{8x}\right)^2}\, dx$$

$$\int_1^3 \left(2x - \frac{1}{8x}\right) dx = (x^2 - \frac{1}{8}\ln x)|_1^3 = 8 - \frac{1}{8}\ln 3$$

4. (6%) Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 + x^2 + 1}{x(x-1)(x^2+x+1)(x^2+1)^2}$$

Do not evaluate the constants.

$$\frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2}$$